Power Domination in triangular grids

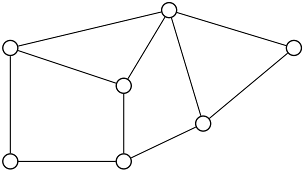
Prosenjit Bose[1] Valentin Gledel[2] Claire Pennarun[3]
Sander Verdonschot[1]

[1]School of Computer Science, Carleton University, Canada
[2]LIRIS, Université Lyon 1, France
[3]LIRMM, CNRS & Univ. Montpellier, France

Domination in graphs

Let $G = (V, E)$ be a graph and $S \subseteq V$.

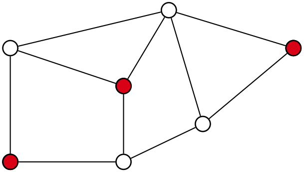
S dominates G if all vertices of G are in S or adjacent to a vertex of S.



Domination in graphs

Let $G = (V, E)$ be a graph and $S \subseteq V$.

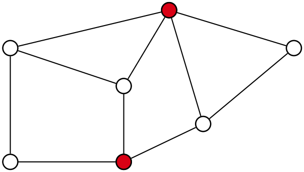
S dominates G if all vertices of G are in S or adjacent to a vertex of S.



Domination in graphs

Let $G = (V, E)$ be a graph and $S \subseteq V$.

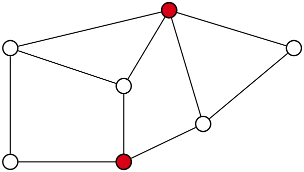
S dominates G if all vertices of G are in S or adjacent to a vertex of S.



Domination in graphs

Let $G = (V, E)$ be a graph and $S \subseteq V$.

S dominates G if all vertices of G are in S or adjacent to a vertex of S.

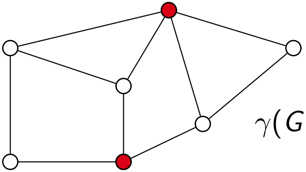


The objective is to find $\gamma(G)$, the size of a minimum dominating set in G

Domination in graphs

Let $G = (V, E)$ be a graph and $S \subseteq V$.

S dominates G if all vertices of G are in S or adjacent to a vertex of S.

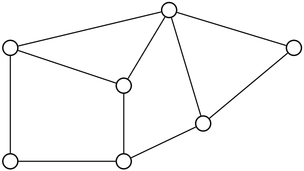


$\gamma(G) = 2$

The objective is to find $\gamma(G)$, the size of a minimum dominating set in G

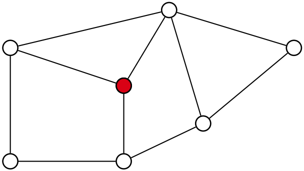
Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



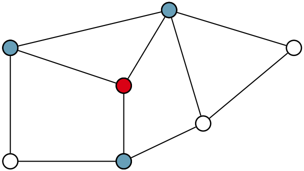
Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



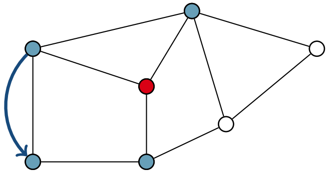
Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



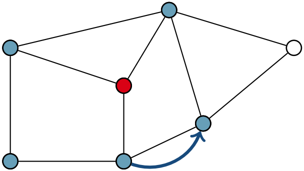
Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



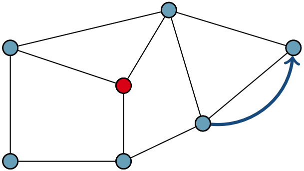
Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



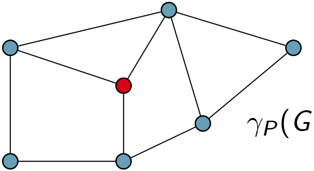
Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



Power Domination

Let $G = (V, E)$ be a graph and $S \subseteq V$.
At first $M = N[S]$. A vertex u *propagates* to a vertex v if $(uv) \in E$ and $N[u] \setminus \{v\} \subseteq M$.
S is a power dominating set of G if at some point $M = V$.



$\gamma_P(G) = 1$

Power Domination

- Introduced in the context of monitoring power grids
  - Mili, Baldwin and Adapa (1990)
  - Baldwin, Mili, Boisen and Adapa (1993)

Power Domination

- Introduced in the context of monitoring power grids
  - Mili, Baldwin and Adapa (1990)
  - Baldwin, Mili, Boisen and Adapa (1993)
- Reformulated in graph terms and proven to be *NP*-complete
  - Haynes, Hedetniemi, Hedetniemi and Henning (2002)

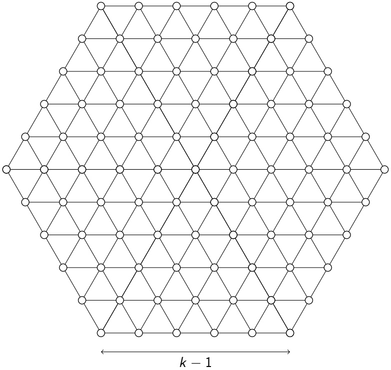
Power Domination

- Introduced in the context of monitoring power grids
  - Mili, Baldwin and Adapa (1990)
  - Baldwin, Mili, Boisen and Adapa (1993)
- Reformulated in graph terms and proven to be *NP*-complete
  - Haynes, Hedetniemi, Hedetniemi and Henning (2002)
- Solved on square grids and other products of paths
  - Doring and Henning (2006)
  - Dorbec, Mollard, Klavžar and Špacapan (2008)
- Solved on hexagonal grids
  - Ferrero, Varghese and Vijayakuma (2011)

Result on hexagonal shaped grid H_k

Theorem

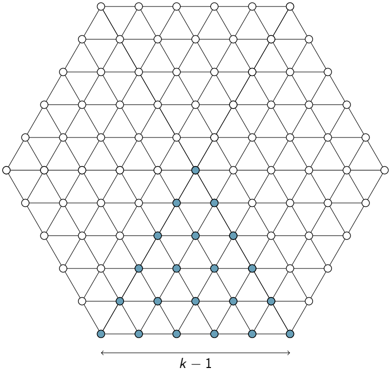
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on hexagonal shaped grid H_k

Theorem

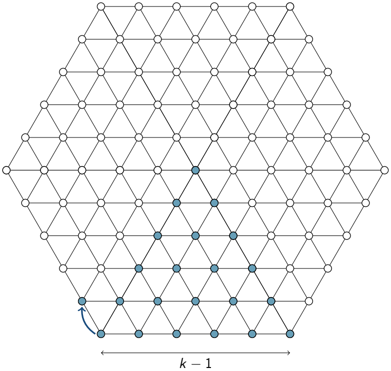
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on hexagonal shaped grid H_k

Theorem

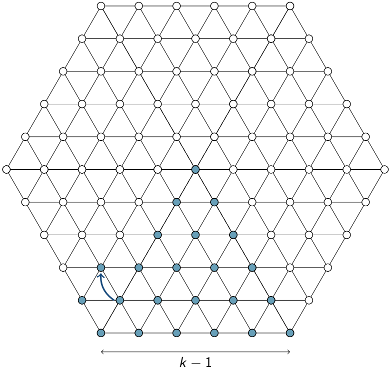
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on hexagonal shaped grid H_k

Theorem

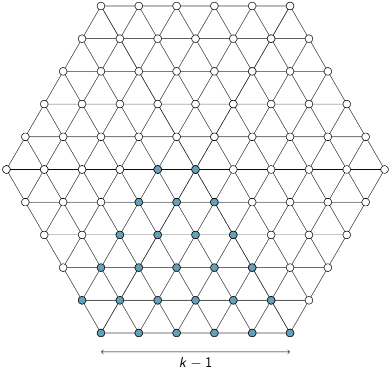
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on hexagonal shaped grid H_k

Theorem

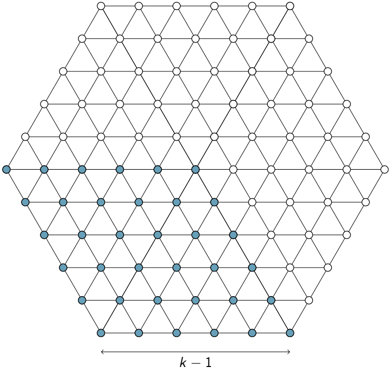
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on hexagonal shaped grid H_k

Theorem

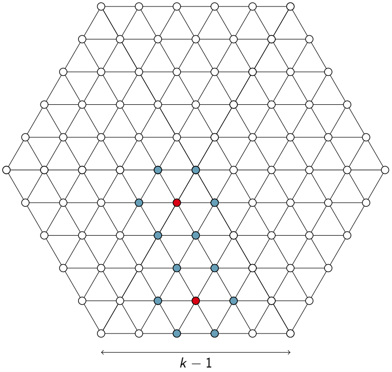
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on hexagonal shaped grid H_k

Theorem

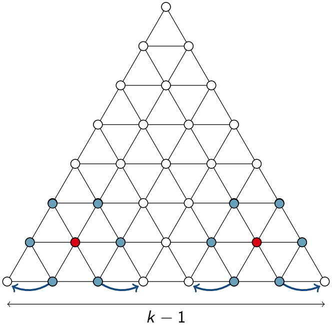
Let H_k be a triangular grid with a regular hexagonal-shaped border of length $k-1$. Then, $\gamma_P(H_k) = \left\lceil \frac{k}{3} \right\rceil$.



Result on triangular shaped grid T_k

Theorem

Let T_k be a triangular grid with an equilateral triangular-shaped border of length $k-1$. Then, $\gamma_P(T_k) = \left\lceil \frac{k}{4} \right\rceil$.



Lower Bound

The proof of the lower bound follows these steps :

Lower Bound

The proof of the lower bound follows these steps :

- We define a function Q on M

Lower Bound

The proof of the lower bound follows these steps :

- We define a function Q on M
- We show that at the beginning $Q(N[S]) \leq 12|S|$

Lower Bound

The proof of the lower bound follows these steps :

- We define a function Q on M
- We show that at the beginning $Q(N[S]) \leq 12|S|$
- We show that Q is non-increasing

Lower Bound

The proof of the lower bound follows these steps :

- We define a function Q on M
- We show that at the beginning $Q(N[S]) \leq 12|S|$
- We show that Q is non-increasing
- We show that at the end, if the grid is fully monitored, $Q(M) = 3k$

Lower Bound

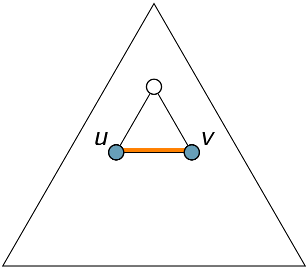
The proof of the lower bound follows these steps :

- We define a function Q on M
- We show that at the beginning $Q(N[S]) \leq 12|S|$
- We show that Q is non-increasing
- We show that at the end, if the grid is fully monitored, $Q(M) = 3k$

This prove that we must have $|S| \geq \frac{k}{4}$

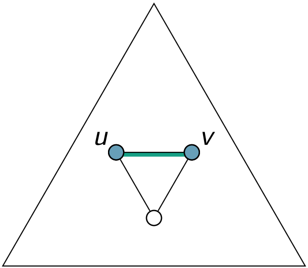
Tip edges and base edges

- An edge (uv) is a tip edge if u and v are monitored but their neighbor in the direction of the tip is not.



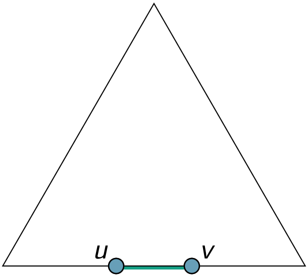
Tip edges and base edges

- An edge (uv) is a tip edge if u and v are monitored but their neighbor in the direction of the tip is not.
- An edge (uv) is a base edge if u and v are monitored but their neighbor in the direction of the base is not.



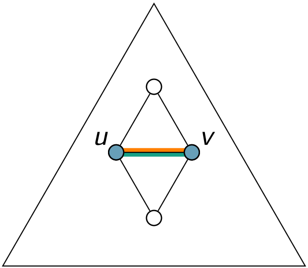
Tip edges and base edges

- An edge (uv) is a tip edge if u and v are monitored but their neighbor in the direction of the tip is not.
- An edge (uv) is a base edge if u and v are monitored but their neighbor in the direction of the base is not.



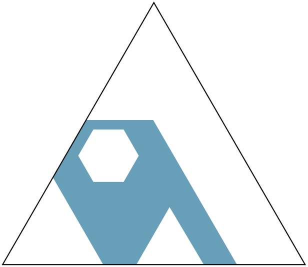
Tip edges and base edges

- An edge (uv) is a tip edge if u and v are monitored but their neighbor in the direction of the tip is not.
- An edge (uv) is a base edge if u and v are monitored but their neighbor in the direction of the base is not.



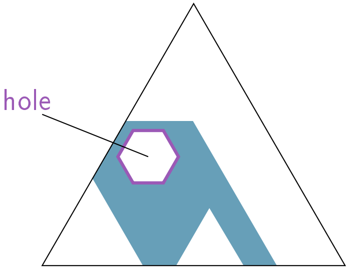
Holes

- A hole is a connected component of $V \setminus (M)$ that does not contain points of the border of the grid.



Holes

- A hole is a connected component of $V \setminus (M)$ that does not contain points of the border of the grid.



The quantity Q

We define the function Q as follows :

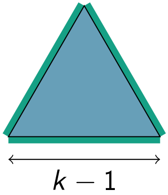
$$Q(M) = 2T + B + 3C - 3H$$

Where :

- T is the number of tip edges
- B is the number of base edges
- C is the number of connected components of M
- H is the number of holes

At the end

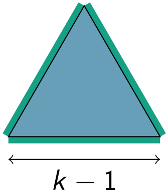
We know the value of Q when all vertices are monitored :



$$Q = 3(k-1) \times 1 + 3 = 3k$$

At the end

We know the value of Q when all vertices are monitored :



$$Q = 3(k-1) \times 1 + 3 = 3k$$

What remains to do :

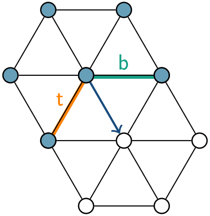
- Proving that Q is non-increasing
- Finding the starting value of Q with respect to S

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



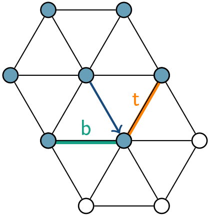
$$Q' = Q - 2 - 1 + \dots$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



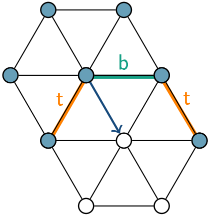
$$
\begin{aligned}
Q' &= Q - 2 - 1 + 2 + 1 \\
&= Q
\end{aligned}
$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



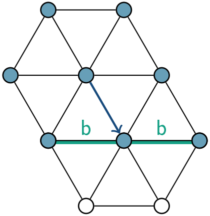
$$Q' = Q - 2 \times 2 - 1 + \dots$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



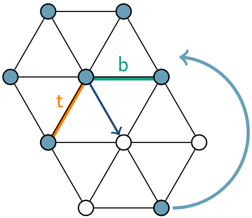
$$Q' = Q - 2 \times 2 - 1 + 2 \times 1$$
$$= Q - 3$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



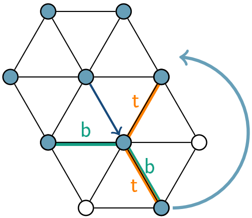
$$Q' = Q - 2 - 1 + \ldots$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



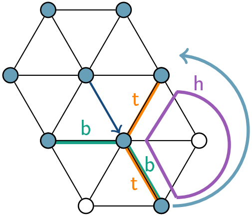
$$
\begin{aligned}
Q' &= Q - 2 - 1 + 2 \times 2 + 2 \times 1 \dots \\
&= Q
\end{aligned}
$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



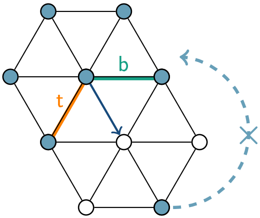
$$Q' = Q - 2 - 1 + 2 \times 2 + 2 \times 1 - 3$$
$$= Q$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



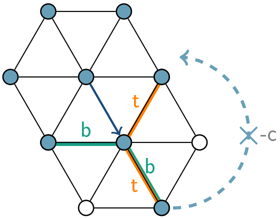
$$Q' = Q - 2 - 1 + \dots$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



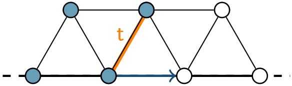
$$Q' = Q - 2 - 1 + 2 \times 2 + 2 \times 1 - 3$$
$$= Q$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



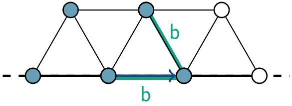
$$Q' = Q - 2 + \ldots$$

Q is non increasing

Lemma

Q does not increase when new vertices are monitored.

We prove this statement by looking at every case:



$$Q' = Q - 2 + 2 \times 1$$
$$= Q$$

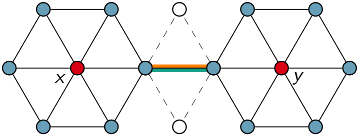
Starting value of Q

Lemma

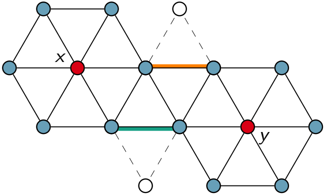
At the beginning, $Q(N[S]) \leq 12|S|$

We suppose here that $N[S]$ is connected.
We define $G_S = (V_S, E_S)$ as follows :

- $V_S = S$
- $(xy) \in E_S$ if x and y form a bridge or a double-bridge :



bridge



double-bridge

Starting value of Q

Lemma

G_S is planar

We can apply Euler's formula:

$$|E_S| - f(G_S) + 1 = |V_S| - c(G_S)$$

Starting value of Q

Lemma

G_S is planar

We can apply Euler's formula:

$$|E_S| - f(G_S) + 1 = |V_S| - c(G_S)$$

We can notice that:

- $|V_S| = |S|$
- $c(G_S) \geq 1$
- $f(G_S) - 1 \leq H$

Starting value of Q

Lemma

G_S is planar

We can apply Euler's formula:

$$|E_S| - f(G_S) + 1 = |V_S| - c(G_S)$$

We can notice that:

- $|V_S| = |S|$
- $c(G_S) \geq 1$
- $f(G_S) - 1 \leq H$

$$|E_S| - H + 1 \leq |S|$$

Use of a discharging method

Lemma

At the beginning, $2T + B \leq 9|S| + 3|E_S|$

We give:

- a weight of 9 to each vertex of S
- a weight of 3 to each bridge and double-bridge

Use of a discharging method

Lemma

At the beginning, $2T + B \leq 9|S| + 3|E_S|$

We give:

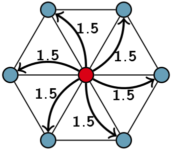
- a weight of 9 to each vertex of S
- a weight of 3 to each bridge and double-bridge

At the end we want:

- A weight of 2 on each tip edge
- A weight of 1 on each base edge
- A non-negative weight on each vertex

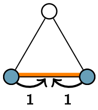
Use of a discharging method

- If u is in S, then it gives a weight of 1.5 to each of its neighbors



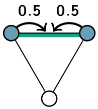
Use of a discharging method

- If u is in S, then it gives a weight of 1.5 to each of its neighbors
- If u is incident to a tip edge, then it gives it a weight of 1



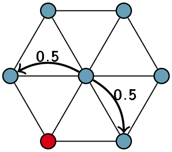
Use of a discharging method

- If u is in S, then it gives a weight of 1.5 to each of its neighbors
- If u is incident to a tip edge, then it gives it a weight of 1
- If u is incident to a base edge, then it gives it a weight of 0.5



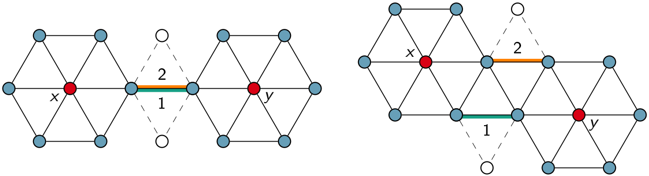
Use of a discharging method

- If u is in S, then it gives a weight of 1.5 to each of its neighbors
- If u is incident to a tip edge, then it gives it a weight of 1
- If u is incident to a base edge, then it gives it a weight of 0.5
- Otherwise, it gives 0.5 to each of its neighbors that it shares with a vertex of S.



Use of a discharging method

- If u is in S, then it gives a weight of 1.5 to each of its neighbors
- If u is incident to a tip edge, then it gives it a weight of 1
- If u is incident to a base edge, then it gives it a weight of 0.5
- Otherwise, it gives 0.5 to each of its neighbors that it shares with a vertex of S.
- Bridges and double-bridges give 2 to their tip edge and 1 to their base edge



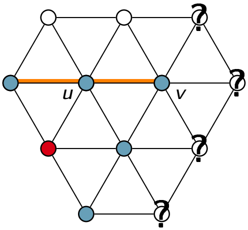
Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

The only possible issue is when a vertex is adjacent to two tip edges.

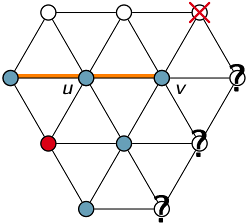


One of the neighbor of v is in S

Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

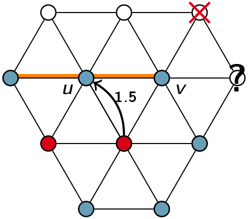
The only possible issue is when a vertex is adjacent to two tip edges.



Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

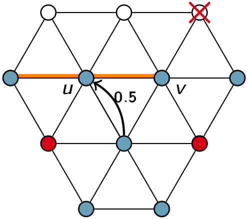
The only possible issue is when a vertex is adjacent to two tip edges.



Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

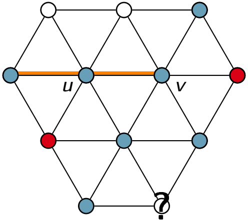
The only possible issue is when a vertex is adjacent to two tip edges.



Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

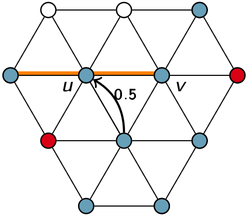
The only possible issue is when a vertex is adjacent to two tip edges.



Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

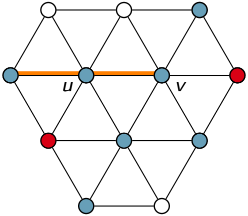
The only possible issue is when a vertex is adjacent to two tip edges.



Use of a discharging method

All tip edges and base edges have the good weight.
We have to make sure that no vertex has a negative weight.

The only possible issue is when a vertex is adjacent to two tip edges.



double bridge

Starting value of Q

We have seen that:

- $2T + B \leq 9|S| + 3|E_S|$
- $|E_S| - H + 1 \leq |S|$

so

$$2T + B \leq 9|S| + 3|S| + 3H - 3$$

this is true for each connected component so

$$Q = 2T + B + 3C - 3H \leq 12|S|$$

Conclusion

- At the beginning, $Q \leq 12|S|$
- At the end, $Q = 3k$
- Q is non-increasing

so :

$$|S| \geq \frac{k}{4}$$

Conclusion

- At the beginning, $Q \leq 12|S|$
- At the end, $Q = 3k$
- Q is non-increasing

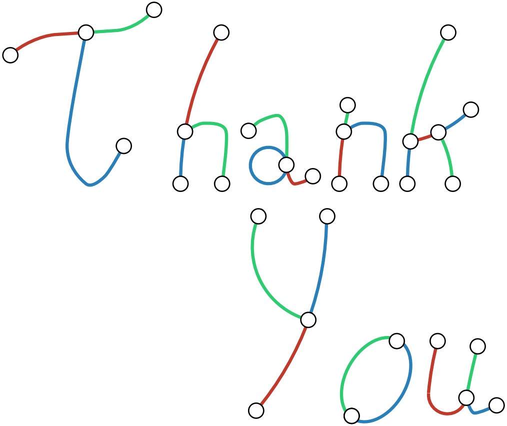
so :

$$|S| \geq \frac{k}{4}$$

This gives us the lower bound and we can reach it so:

Theorem

$$\gamma_P(T_k) = \left\lceil \frac{k}{4} \right\rceil$$


Thank
You